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LETTER TO THE EDITOR

Collective behaviour in one-dimensional locally coupled map lattices

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Abstract. We observe true and quasiperiod-4 collective evolution of average values in one-dimensional lattices of logistic maps with variable coupling range. The phenomenon appears to be quite prevalent for local coupling beyond nearest neighbours. We find that in one dimension it is easy to visualize the structure of the four (quasi)states; these appear to be quite stable to disturbances such as domain reversals.

Collective behaviour of spatially coupled, strongly nonlinear systems has attracted recent interest. Indeed, droplet-type arguments by Bennett *et al* (1990) seemed to suggest that the allowed patterns of stable collective states in two- and higher-dimensional models are rather limited. Recent numerical evidence by Chaté and Manneville (1991, 1992a, b), however, established cases of quasiperiodicity-3 and other similar collective states in certain high-dimensional ($d = 4,5$) deterministic cellular automata. These collective effects were inconsistent with the 'droplet' ideas. No phenomenological or analytical theory of such collective states is available to date. Further numerical effort has concentrated on systematic studies of such higher-dimensional models, including stability to noise and perturbations, sensitivity to initial conditions, and, to the extent it is quantifiable, the universality of the observed patterns of self-organization; see Gallas *et al* (1991, 1992). A similar pattern of behaviour in three dimensions was found and studied by Hemmingsson (1991).

Emergence of collective behaviour in high-dimensional coupled-map lattices has also been established recently, by Chaté and Manneville (1992a, b). Many aspects of the patterns observed and their classification were similar to those found earlier for discrete-spin cellular automata. The emphasis in both cellular automata and coupled-lattice work by Chaté and Manneville (1991, 1992a, b) was on the competition between the tendency to disorder in complex dynamical systems with short-range couplings, and the mean-field-like tendency to uniform behaviour typical of high-dimensional models. Thus, collective effects were sought in high dimensions. It was realized, however, that the actual pattern of the return map of the order parameter, studied extensively by Chaté and Manneville (1991) and Gallas *et al* (1991, 1992) for the cellular automata case, was only loosely, if at all, related to the features of the mean-field map.

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In this letter we consider instead one-dimensional lattices of logistic equations in which the range of coupling is allowed to vary. The idea of scaling range with system size has been used recently (Owczarek *et al* 1992) to find the necessary conditions for ordering in one-dimensional spin systems. As interaction range increases, models in one dimension may have the mean-field tendency to uniformity, even for a fixed coupling range. Coupled map lattices in one dimension were studied extensively for nearest-neighbour couplings (Crutchfield and Kaneko 1988, Kaneko 1989a, b, 1990). Quasiperiodic states with periods near 2, 4, and 8 were noted for some parameter values (Kaneko 1989b, 1990). However, the emphasis of the previous studies was on quantifying chaotic behaviour.

The main finding of this letter is the existence of periodic and quasiperiodic (in the sense of periodic with irrational period) collective behaviour in such lattices, which seems to be attributable to the interplay of uniformizing mean-field averaging as the coupling range increases, with the randomizing effect of the logistic map in the parameter range of chaotic behaviour of the latter. The advantages of the one dimensional systems with variable coupling range are: (1) the system must become mean-field when the coupling range is of the order of the system size, and (2) the spatial structure (if any) of the (quasi)ordered states and the recovery of disturbances are easier to visualize than in higher dimensions. A difference is of course that the droplet arguments are not relevant to one dimension. However, recent $d > 1$ studies put such arguments in question anyway. Furthermore, the observed self-organized states in one dimension seem spatially ordered (see below) whereas the high-d collective states were disordered to the extent this property could be quantified by studying lower-dimensional cross-sections. Another difference is that, at least for the smaller lattice sizes considered in this letter, we do not find 'thermodynamic' noise like that observed in higher dimensions.

The evolution rule for the system of L maps arranged in a periodic linear array is as follows. Successive averaging and iterating steps are taken. First, the local variables x_i, ranging between zero and one, are averaged over r neighbours from both sides according to

$$x'_i = (2r+1)^{-1} \sum_{j=i-r}^{i+r} x_j(t). \tag{1}$$

Next, each locally-averaged value is iterated with the well known logistic map by

$$x_i(t+1) = a x'_i(1 - x'_i) \ . \tag{2}$$

We now define a collective order-parameter variable by

$$X(t) = L^{-1} \sum_{j=1}^{L} x_j(t) \ . \tag{3}$$

Simulations for a number of lattice sizes L, coupling ranges r, nonlinearity parameters a and initial conditions have been performed. We first describe typical results before concentrating on a specific set of parameters.

From (1)–(3), one deduces that the behaviour of maps coupled with $r = (L/2)-1$ is exactly mean-field, and should therefore coincide with the behaviour of a single logistic map in every respect: the time-evolution of the system, the long-time values and distributions of $X(t)$, and the form of the return map $X(t+1)$ versus $X(t)$. The long-time distribution of iterates of x (often plotted versus a and known as a bifurcation diagram) is well known for a single map (see Grossmann and Thomae 1977, Bergé *et al* 1984).

One sees indeed that as r increases the bifurcation diagram (in the range of interest $3 \leqslant a \leqslant 4$) approaches that of a single map; it does so fairly systematically for the lower range of a, but the behaviour for $a > 3.6$ changes somewhat erratically up to $r \sim L/4$. For larger r the bifurcation diagram appears to be identical to the single-map diagram for all a.

A systematic search over the parameter space of (a, L, r) would be unfeasible. We now describe in detail a particular example of (quasi)periodic behaviour, followed by a discussion of how prevalent these phenomena are.

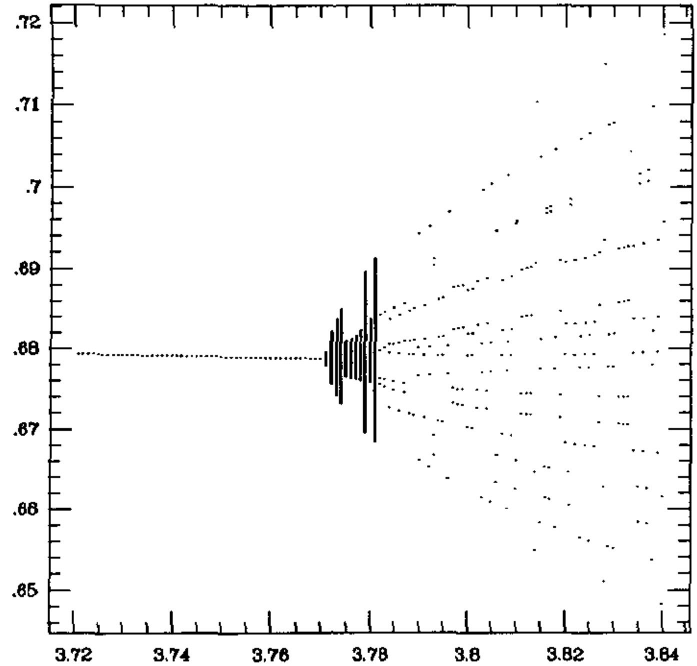

Figure 1. Long-time iterates of equations (1), (2) with $L = 81$, $r = 4$ versus nonlinearity parameter a: detail for $3.72 \leqslant a \leqslant 3.84$.

The case we consider in detail corresponds to $L = 81$, $r = 4$. Figure 1 corresponds in particular to parameter values $3.72 \leqslant a \leqslant 3.84$. Return maps $X(t+1)$ versus $X(t)$ yield noisy versions of the parabola (2) for values such as $a = 3.91$, and therefore correspond more or less to the mean-field limit. The more interesting cases correspond to the transition values between fixed points and period-4 cycles as seen in figure 1: the return map of X for $a = 3.78$ is given in figure 2.

The quasiperiodic behaviour of the map for $a = 3.78$ is quite evident; its nearest integer period has been investigated with the 'polar' variable θ, introduced by Chaté and Manneville (1991), a measure of the angular position of a point in the return map trajectory with respect to an arbitrary point near the centre of this trajectory. A

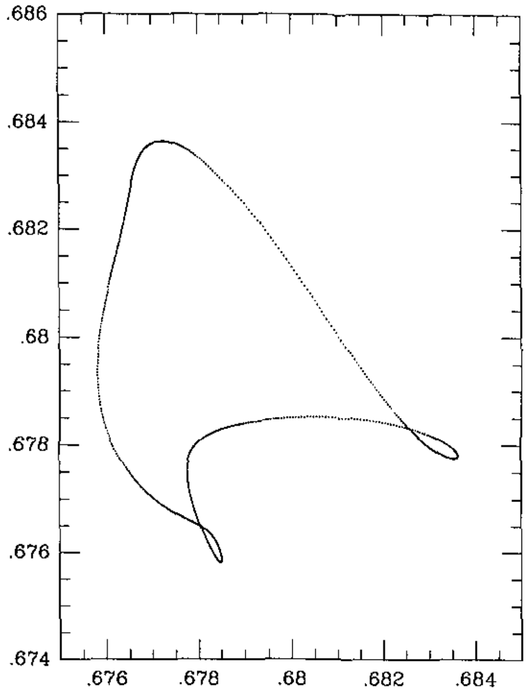

Figure 2. Return map $X(t+1)$ versus $X(t)$ for $L = 81$, $r = 4$ and $a = 3.78$; times shown are $997\,000 \leqslant t \leqslant 1\,000\,000$.

plot gives four distinct blobs more or less coincident with the line $\theta^{t+4} = \theta^{t}$. This confirms the quasiperiod-4 nature of the system. For $a > 3.782$ the return map for X collapses to four points.

The spatial structure of the (quasi)periodic states is easy to visualize. We show in figure 3 the values x_i versus i for four consecutive time steps. The parameter values are the same as for figure 2. Each global state consists of smoothly varying values x_i, forming three oscillating domains of width 27 each, which move by half a wavelength after each time step, reminiscent of those reported in globally coupled map lattices (Kaneko 1986), where no quasiperiodic behaviour was reported. These repeat following the same sequence, exactly in the periodic case and almost exactly in the quasiperiodic case. The long-time average of each of the four states is also quite well-defined, consisting of an almost perfect sinusoid of wavelength 27 sites, average value 0.68 and amplitude 0.04.

A few remarks are in order for this quasiperiodic trajectory: (1) it seems to be quite independent of the initial condition; (2) the times considered are of order 10^6 time steps, as $t \gg L^2$ we believe that this is *not* a transient phenomenon; (3) the long-time states are considerably stable to major disturbances. For instance, we took 27 sites of one of the states depicted in figure 3, and replaced them with the (antinodal) values corresponding to the previous time step; the system healed to its original state after about 500 time steps.

We discuss now the prevalence of this phenomenon. First of all, for any pair (L, r) it is localized over a very narrow window of a, as exemplified in figure 1. Therefore, it is quite easy to miss it. For $L = 81$ we observed it for values $r = 7, 12, 13, 14$. We also observed it for lattice sizes of 100 ($r = 16$), 107 ($r = 9$), 128

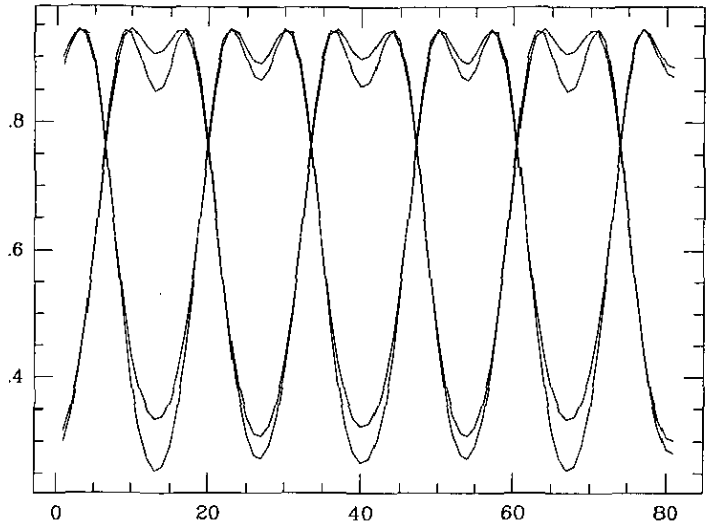

Figure 3. Long-time values of x, for four consecutive time steps, superimposed; parameter values same as in previous figure. Quasiperiodic behaviour: structures almost repeat. Periodic behaviour: structures repeat exactly.

($r = 17$), 243 ($r = 6$), and for lattices as large as $L = 20\,000$. It always seems to happen during a forward fixed-point to period-4 transition; the corresponding values of a are quite variable, between 3.7 and 3.9. We believe that the quasiperiodic behaviour is caused by a delicate interplay between the trajectory-separating tendency of the single map (2) for certain chaotic values of a, and the effect of averaging (1). Finally, only for large enough lattices ($L \sim 1000$) does thermodynamic-like noise appear.

As remarked in the introductory discussion, there is no evidence to suggest that the collective states observed in one dimension are similar to those found in higher-dimensional models. In fact, the competition between the mean-field averaging and the logistic map tendency for chaotic spread in values, is the only common feature that can be identified at this stage. Our large-size results indicate that, while deterministic 'noise' appears, collective soliton-like states in one dimension persist in the 'thermodynamic limit'. Therefore, the disordered (in space) pattern of the higher-d collective states is not shared by the one-dimensional models.

In summary, in this letter we have reported a numerical exploration of the behaviour of a one-dimensional array of nonlinear maps, in which the range of coupling is allowed to vary up to mean-field. The 'route' to mean-field is therefore different than that explored by previous authors. We observe collective periodic and quasiperiodic behaviour, which seems to be present for coupling ranges greater than one but smaller than $\sim L/4$. The quasiperiod-4 behaviour usually happens during a forward transition from fixed point to period 4, and the values of a at which quasiperiodic behaviour happens are fairly narrow, indicating a delicate balance between the averaging effect of the coupling and the randomizing effect of the chaotic maps.

One of the advantages of a one-dimensional model is that the structure of the (quasi)ordered states can be easily visualized, as in figure 3. An interesting feature is that the collective states appear to be formed from well-defined, oscillating domains. This is different from the disorder seen in higher dimensional systems. While the relation between the phenomena observed here and those observed in higher dimensions is not clear, our results suggest that cooperative behaviour may be caused not only by conventional thermodynamic phases, but also by coherent soliton-like structures. It should be mentioned that Kaneko (1990) has observed soliton-like *propagating waves for maps with nearest-neighbour 'antiferromagnetic' coupling*.

Both the similarities and differences with higher-dimensional models should be helpful in understanding in a unified manner how collective behaviour arises.

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